Problem Set – Internet Economics course 2010/11

Submission date: February 1st,
in Liad’s tray (Economics department, 2nd floor).

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Instructions:

• Please print the problem set and answer the questions in the allotted space. Please do not add extra pages.

• Please answer all questions. All questions have equal weight.

• When you are asked to write a number, please only write a number with no supporting text.

• When you are asked to write text, be brief. Write in Hebrew.

• Q. 1 is about single-item auctions, discusses in classes 1 and 2.

• Q. 2 is about VCG mechanisms, discussed in class 3.

• Q. 3 is about diffusion in social networks, discusses in class 7 and 8.

• Q. 4 is about sponsored search auctions, discusses in class 5.

Good luck!
1 Question 1: affirmative-action auctions

A seller is selling a single good to a set of \( n \) bidders. From the \( n \) bidders, \( n_B \) are boys and \( n_G \) are girls (\( n_B, n_G > 0, n_B + n_G = n \)). Consider the following auction (”affirmative-action auction”):

- Each bidder submits a bid for the item.
- Let \( b^* \) be the boy with the highest submitted bid, and let \( g^* \) be the girl with the highest submitted bid.
- The auctioneer tosses a coin, and with probability \( \frac{1}{2} \) the boy \( b^* \) wins and with probability \( \frac{1}{2} \) the \( g^* \) wins.
- If \( b^* \) wins, he pays the bid of the girl \( g^* \). If \( g^* \) wins, she pays the bid of the boy \( b^* \).

Prove or disprove: \textit{the affirmative-action auction is truthful in dominant strategies.}

Remark: Disproving the claim requires showing a scenario where mis-reporting the true value can benefit some bidder. Proving the claim requires a set of arguments (like the arguments given in class for the second-price auction) showing that a bidder will never benefit from lying.
2 Question 2: Weaknesses of VCG mechanisms

A seller is selling two items, $a$ and $b$. Two bidders are interested in buying the items:

- Bidder 1 is willing to buy the two items together for $201, but is willing to spend just few dollars for each item separately. That is, $v_1(ab) = 201, v_1(a) = 1, v_1(b) = 3$.

- Bidder 2 is willing to buy each item for $200, and a little more for the whole bundle. That is, $v_2(ab) = 202, v_2(a) = 200, v_2(b) = 200$.

1. Describe an efficient allocation of the two items?
   **Answer:** Bidder 1 receives items __________ and Bidder 2 __________

2. In a VCG auction, how will the items will be allocated? How much each one of the bidders pays in a VCG auction?
   **Answer:** In the VCG auction,
   Allocation: Bidder 1 receives items __________ and Bidder 2 __________
   Payments: Bidder 1 pays __________ and Bidder 2 pays __________

3. A third bidder enters the market. The third bidder has preferences identical to bidder 2. That is, $v_3(ab) = 202, v_3(a) = 200, v_3(b) = 200$. How will the items be allocated now in a VCG auction, and what are the payments for the bidders?
   **Answer:**
   Allocation: Bidder 1 receives items __________, Bidder 2 __________ and Bidder 3 __________.
   Payments: Bidder 1 pays __________, Bidder 2 pays __________ and Bidder 3 pays __________.

4. An auction is *revenue-monotone* if expanding the market (that is, adding bidders to the market) can only increase the seller's revenue. Given your answer to the previous item, is the VCG auction monotone?
5. Shill bidding (or "false-name" bidding) is when a bidder submits additional bids as a fake bidder.

Consider now the original two bidders 1 and 2. Given your answers to the previous items, show that Bidder 2 can benefit from shill bidding.

**Answer:** Bidder 2 can benefit from shill bidding by (explain your answer):
3 Question 3: Diffusion in Social Networks

Suppose that initially everyone is using behavior B in the social network in Figure 1, and then a new behavior A is introduced. This behavior has a threshold of $q = 1/2$: any node will switch to A if at least $1/2$ of its neighbors are using it.

1. Are there two local bridges in the graph that originate from the same node? Explain.

   

   

   

   

   

2. Find a set of three nodes in the network with the property that if they act as the three initial adopters of A, then it will spread to all nodes. (In other words, three nodes who are capable of causing a cascade of adoptions of A.)

   Answer: The three nodes are _____________________________

3. Find three clusters in the network, each of density greater than $1/2$, with the property that no node belongs to more than one of these clusters.

   Answer: The clusters are _____________________________
   (Please also circle the clusters in the figure.)
4. How does your answer to (c) help explain why there is no set consisting of only two nodes in the network that would be capable of causing a cascade of adoptions of A? (I.e., only two nodes that could cause the entire network to adopt A.)

**Answer:**

Figure 1: This is the network discussed in Question 3.
4 Question 4

Suppose a search engine has two ad slots that it can sell, and seller is using the Generalized Second-Price auction (GSP). Slot 1 has a click-through rate of 0.14 and slot 2 has a clickthrough rate of 0.08. There are three advertisers who are interested in these slots. Advertiser $x$ values clicks at $18$ per click, advertiser $y$ values clicks at $13$ per click and advertiser $z$ values clicks at $9$ per click.

Compute a set of bids for the advertisers that are an equilibrium in the normal-form game created by the auction (as presented in class).

Explain your answer.

Answer: In equilibrium,

Advertiser $x$ bids ________________.

Advertiser $y$ bids ________________

and Advertiser $z$ bids ________________.

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